Self-Imposed Limits to Arbitrage

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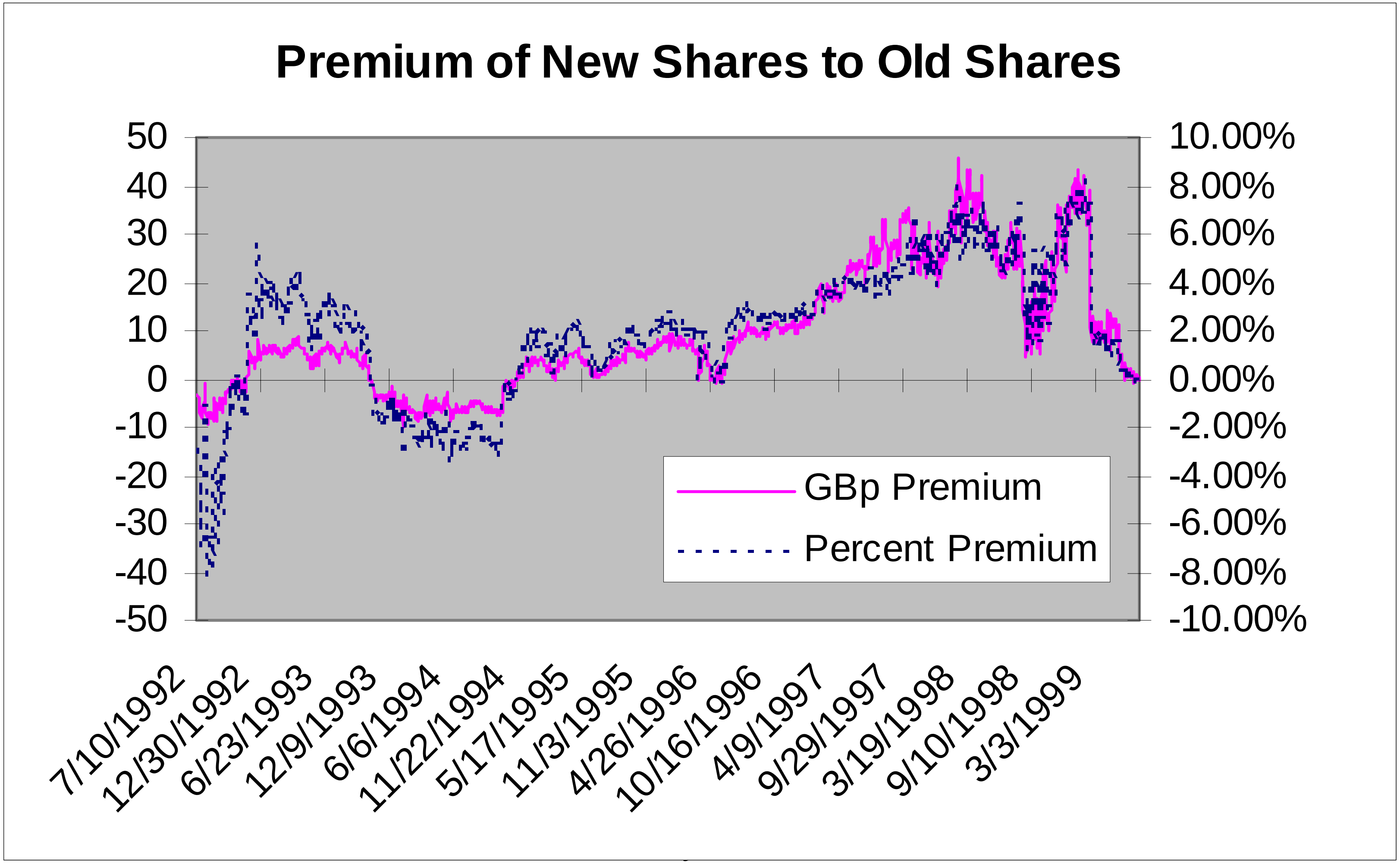
HSBC Share Class Discrepancy

July 10, 1992 – July 2, 1999

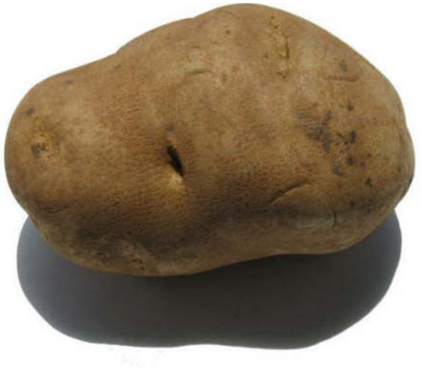


The (John) Law of One Price

- John Law (1671-1729), Scottish economist
- **LOOP: "In an efficient market, all identical goods must have only one price."**
- Law's Louisiana shares: guaranteed by the king and exchangeable into silver
  - 1719: shares cost 500 livres
  - 1720: shares cost 10,000 livres (though went as high as 18,000)
  - 1721: shares cost 300 livres
- How did he do it? "Marketing."

“In an efficient market, all identical goods must have only one price.”

LOOP vs. LTA



- You see two potatoes:
  - One costs $1.00, the other $1.02
- LOOP -- Law of One Price:
  - ***All*** buyers should buy the cheap one.
  - ***All*** sellers should sell at the high price.
- LTA -- Limits to Arbitrage:
  - Some buy too high and some sell too low.
  - Arbitrageurs try to set it right.
    - But they can’t borrow potatoes.

(External) Limits to Arbitrage

- Hard, costly, or illegal to borrow
  - 3Com/Palm spinoff: no Palm shares available
- Noise trader risk
  - Royal Dutch/Shell: home bias
- Transactions costs
  - Thai local vs. foreign shares: need local bank
- Performance-based arbitrage
  - LTCM: hedge fund investors chase performance so capital is unavailable exactly when it is needed most
- To date, it has been possible to explain all discrepancies with external limits to arbitrage.

The Problems with Other Pairs

Type of Pair	Twins	Stubs
Example	Royal Dutch-Shell	3Com/Palm
Complications	Different countries, currency, trading times, tax treatment, and index membership	Hard to borrow, different index membership and industry
Limit to Arbitrage	Noise trader risk (home bias)	Short sales constraints

HSBC is a serendipitous example. Best of both worlds:

– Unlike twins, traded on the same exchange in the same currency at the same time, with identical tax treatment and index membership. **No home bias noise trader risk.**

– Unlike stubs, both stocks were easy to borrow and both were in the same indices and of course in the same industry. **No short sales constraints.**

But HSBC is still mispriced, so perhaps the mechanism driving the HSBC mispricing is also the *real* reason behind the mispricings of other twins and stubs.

Substantial Discrepancy

- Premium of New Shares to Old Shares
  - Average: 1.9%; Average Absolute Value: 2.9%
  - Peak: 8.3%; 95th Percentile: 6.4%; 90th: 5.5%; 85th: 4.8%
- Average Total Market Cap: About £40 billion
  - On peak discrepancy dates, mispricing range exceeds £3 billion
- Analysts found no justification for discrepancy
  - SBC Warburg, 1997: "Institutions choosing to pay the 56p premium for the STG stock appear to be wasting the money and we recommend they buy the HK$ stock and enhance the yield."
  - BZW Securities, 1997, among others, concurred
- Amount of capital used and "wasted" by noise traders
  - Total capital committed to more expensive share: £60 billion
  - Total excess cash spent on the more expensive share: £2 billion

Issuance of New Shares

- June 25, 1992
  - HSBC acquired Midland Bank stock-for-stock.
  - Could not issue HK$-par value HSBC stock to pay for GBP-par value Midland Bank stock.
  - So they created a new class of shares ("New" shares) with a 75p-par value.
  - The two share classes were intended to be identical.
  - But because of par values, they could not be fungible.
  - From the Listing Particulars:

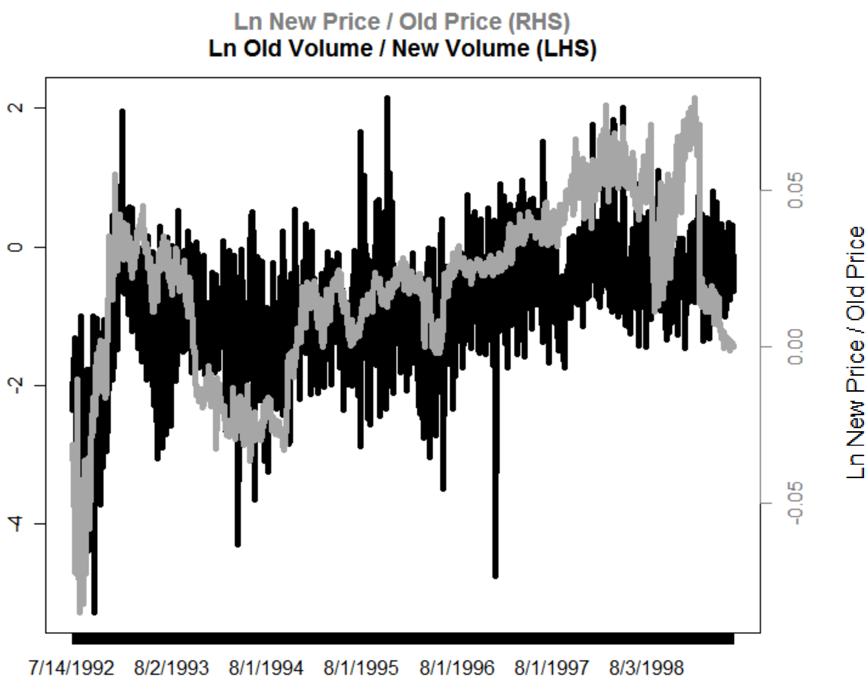
    "The new HSBC Holdings shares to be issued pursuant to the Offer will be issued credited as fully paid and will rank pari passu in all respects and have identical rights with the existing HSBC Holdings shares, including the right to receive in full all dividends and other distributions declared, made or paid hereafter."

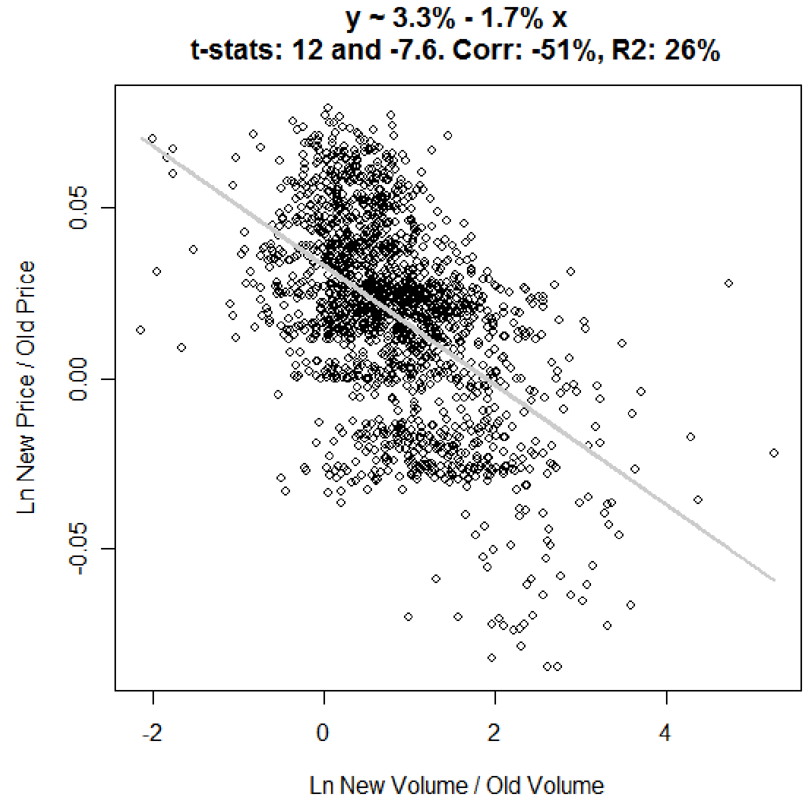
HSBC New and Old Shares

	New	Old
Par Value	75GBp	HK$10
Index Membership (FTSE 100, All Share)	Yes	Yes
Retail Ownership by Individuals (1992)	14.36%	24.14%
Retail Ownership by Individuals (1993)	9.01%	20.05%
Average Bid-Offer Spread (basis points)	40.3 [0.7]	41.2 [0.7]
Average Daily Traded Value	£39,150,000	£23,870,000
Average Annual Turnover	320%	84%
Correlation of Log Price with Log Volume	-0.12	0.33
Standard Deviation of Log Daily Returns	1.97% [0.03]	1.99% [0.03]
Weekly Beta to FTSE 100 Index	1.62 [0.09]	1.60 [0.09]
Fama-French: Loading on Mkt-Rf	1.33 [0.09]	1.38 [0.09]
Fama-French: Loading on SMB	0.60 [0.11]	0.69 [0.11]
Fama-French: Loading on HML	0.75 [0.13]	0.79 [0.14]

Correlation between daily returns of New and Old shares: 97%

Puzzle 2: The Volume Discrepancy





Self-Imposed Limits to Arbitrage

- Solves all the problems
- Pricing puzzle
  - Why is there a difference in price at all?
  - How can the premium become a discount? And why at those particular times?
- Volume puzzle
  - Why is the relative volume negatively correlated with the relative price?
- BONUS: Also explains other pairs!

Why Impose Limits on Yourself?

- What would keep arbitrageurs from trading as much as possible as fast as possible?
  - Internal restriction on position size
    - "No more than 5 days average volume."
  - Internal restriction on volume participation
    - "No more than 15% of average volume."
- Why place such restrictions on yourself?
  - Agency or self-control purposes
  - Marketing to investors the benefit of fixed risk limits
  - Ability to liquidate in an orderly fashion

Formal SILTA Model

Choose *t** to maximize:

$$\max_t \sum_{s=1}^{t-1} \frac{s}{t} Np(1-p)^{s-1}\left(E_0(D_s) - f\left(\frac{s}{t}N, s\right)\right) + N(1-p)^t\left(E_0(D_t) - f(N,t)\right) \quad (1)$$

For industry-standard square-root market impact costs, solve numerically:

$$\frac{N}{(1-p)pt}\left(\left(A(t,p) + C(t,p)\right)D_0 - \left(B(t,p) + C(t,p)\right)\sqrt{\frac{N}{t}}\right) = 0 \quad (3)$$

where:

$$A(t,p) \equiv 2\left(1-(1-p)^t\right)(1-p) \quad (4)$$

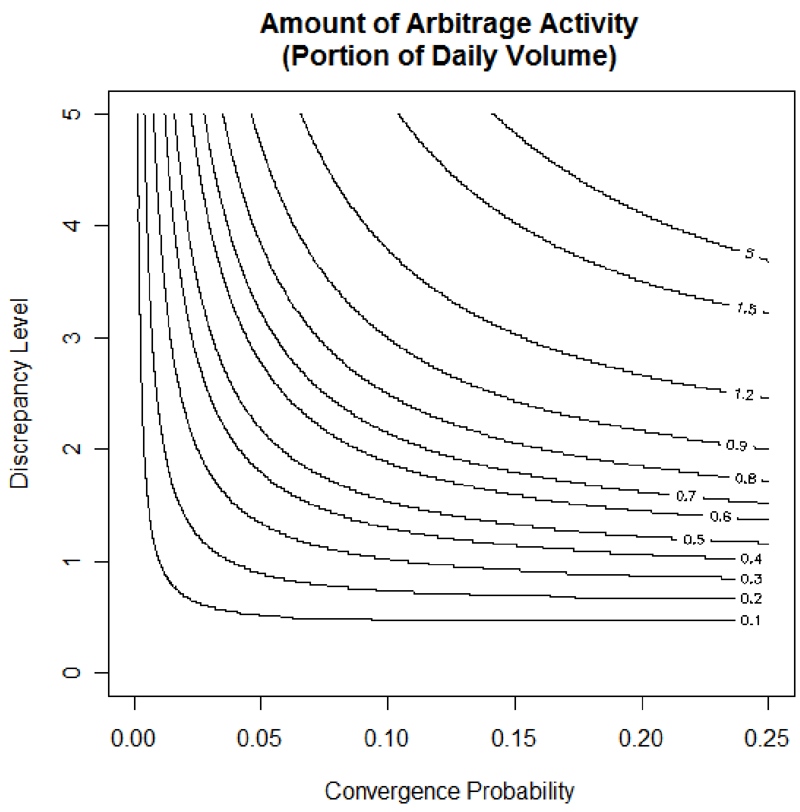
$$B(t,p) \equiv 3(1-p) - (1-p)^t\left(3 + p(pt-3)\right) \quad (5)$$

$$C(t,p) \equiv 2(1-p)^t t\left(1 + p(pt-1)\right)\ln(1-p) \quad (6)$$

SILTA and the Pricing Discrepancy

SILTA predicts that 1) arbitrageurs will not immediately establish their entire position and 2) will trade more as the discrepancy widens.

Formally, arbitrageurs maximize their total expected profit, which depends on transaction costs, the probability of overnight convergence, and the current value of the pricing discrepancy.



SILTA and the Volume Discrepancy

- **E** is the expensive share, **C** the cheap one
- Suppose **E** tends to have more volume.
  - This is typically the case and is true for HSBC.

$$\frac{V_E}{V_C} > 1$$

- When the relative price goes up, so **E** is even more expensive, arbitrageurs trade more.
  - Why? Self-imposed limits to arbitrage.

$$k \uparrow$$

- When arbitrageurs trade more, relative volume goes down.
  - Why? Because they essentially trade equal shares of both stocks.

$$\frac{V_E + k}{V_C + k} \to 1$$

SILTA Successes

- SILTA explains both the pricing and volume discrepancy in HSBC, which is itself a unique example of dual shares.
- Does it also explain other pairs? YES!

Pair	$\beta = \rho$	Std. Err.	t-Stat	$\mathbf{R}^2$
HSBC New-Old	-22%	5%	-4.2	5%
3Com/Palm	-43%	15%	-2.9	19%
RD-Shell '02-'07	-25%	7%	-3.4	6%
Unilever '02-'07	-23%	6%	-3.8	5%
Reed-Elsevier '02-'07	-9%	4%	-2.2	1%
Creative/Ubid	13%	19%	+0.7	2%
HNC/Retek	10%	9%	+1.1	1%
DaisyTek/PFSWeb	12%	7%	+1.8	1%
Metamor/Xpedior	-21%	13%	-1.6	4%
Methode/Stratos	-3%	10%	-0.3	0%

Can SILTA Tell Us Even More?

- SILTA: ***Θ = f(N,p,D)***
- ***Θ:*** The portion of daily volume that is arbitrage activity.
- ***N:*** The primary parameter to the model is a position limit.
  - "We don't want to hold more than *N* days volume in any stock."
  - ***N*** is something like 5 days, per hedge fund.
- ***p:*** Probability of overnight convergence.
  - ***p*** is very low, something like 1% or less.
- ***D:*** The current discrepancy, or relative price.

SILTA Regressions

- Standardize relative prices and volumes for ease of comparison among pairs.
- SILTA predicts a negative relation between relative price and relative volume:

$$Z\left[\ln \frac{P_1}{P_2}\right] = a + bZ\left[\ln \frac{V_1}{V_2}\right]$$

- But we can do regressions on model outputs for an even tighter prediction:

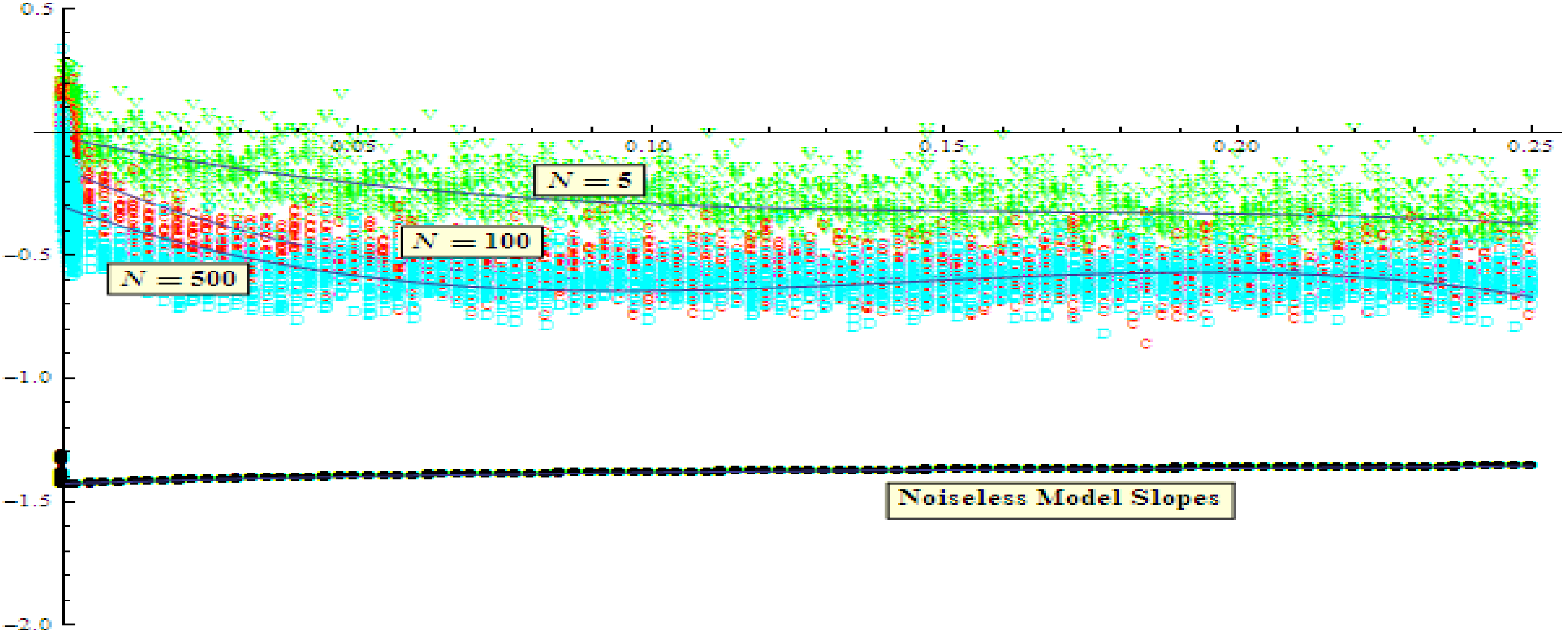
$$D = a + bZ\left[\ln\left(1 + \frac{\chi}{1+\theta}\right)\right]$$

- where: $\frac{V_1}{V_2} = \frac{S_1 + \theta S_2}{S_2 + \theta S_2} = \frac{S_1/S_2 + \theta}{1+\theta} = 1 + \frac{\chi}{1+\theta}$

- and we let $1+\chi$ be a randomly distributed lognormal variable with parameters matching empirical estimates.

SILTA's Implied Parameters

- For given ***p*** and ***N***, take 200 ***D***'s, calculate the 200 matching ***Θ***'s, and regress, using estimated ***μ*** and ***σ*** from the volume data.
- Plot the resulting regression coefficients and smooth with a cubic fit.
- Now for a given ***p*** and a given actual regression coefficient, we can find the matching ***N***.



Implied Probability of Convergence

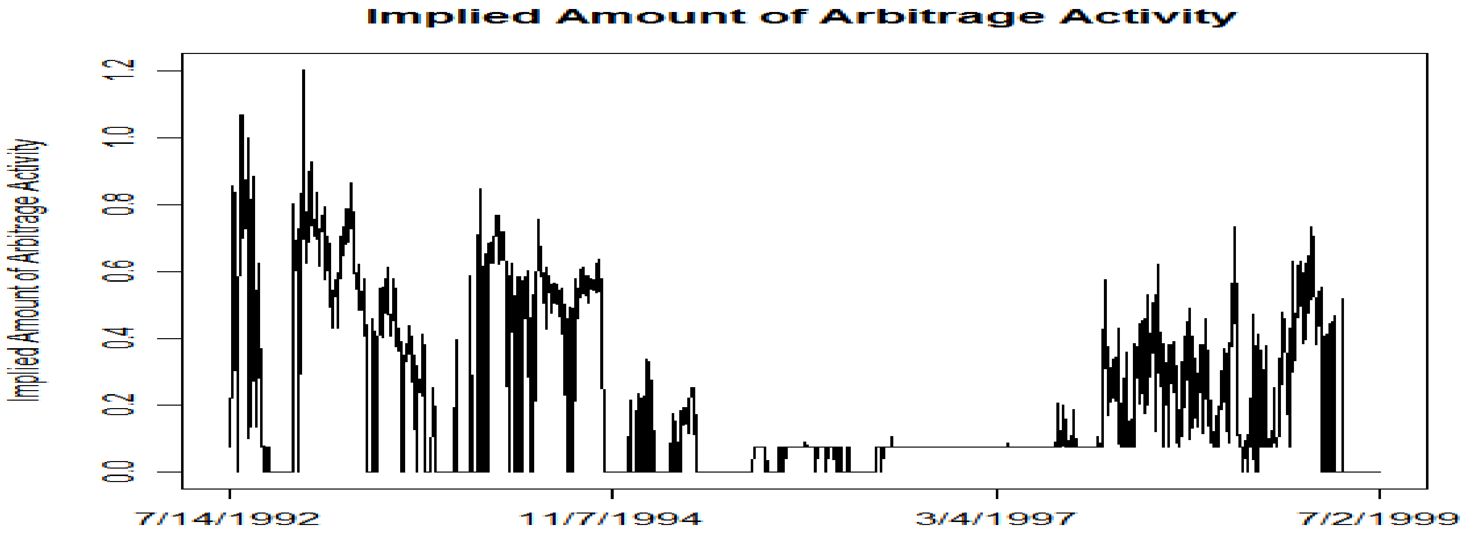
- Difficulties:
  - Impossible to tell ahead of time.
  - Probably different for each arbitrageur.
- Some rough estimates:
  - Solve for daily ***p*** such that convergence has a cumulative 50% probability within six months:

$$(1-p)^{126}=0.50 \Rightarrow p=0.55\%$$

  - More specifically, use the actual number of crossing by HSBC to suggest that ***p*** = 0.95%.

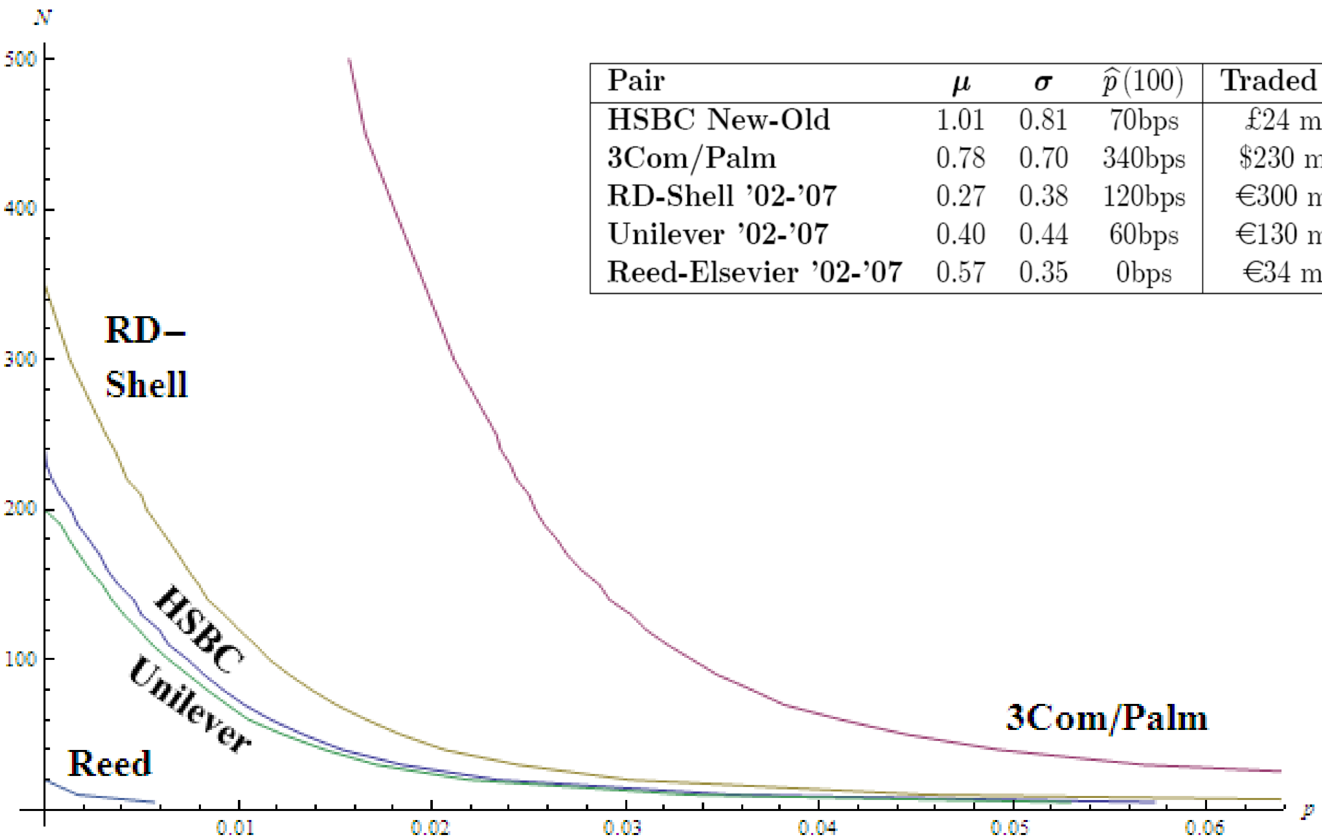
How Much Arbitrage Activity?

- Given ***p*** = 0.95% and a regression coefficient of -22%, the implied ***N*** = 77.
- Using those values of ***p*** and ***N***, and the daily ***D***, calculate the daily amount of arbitrage activity.



Implied ***p*** and ***N***

- Do the same thing as before with SILTA regressions, but now plot ***N*** as a function of ***p***.



Pair	μ	σ	$\widehat{p}(100)$	Traded Value
HSBC New-Old	1.01	0.81	70bps	£24 million
3Com/Palm	0.78	0.70	340bps	$230 million
RD-Shell '02-'07	0.27	0.38	120bps	€300 million
Unilever '02-'07	0.40	0.44	60bps	€130 million
Reed-Elsevier '02-'07	0.57	0.35	0bps	€34 million

Summary and Conclusions

1. HSBC pair had a multi-billion dollar discrepancy
   - Pricing puzzle: swings up to 8% both ways
   - Volume puzzle: relative price higher when relative volume lower
2. Self-imposed limits to arbitrage explains both puzzles
   - Arbitrageurs choose to have position and trading restrictions
   - Arbitrageurs trade equal volume in both shares of a pair, which brings relative volume closer towards one
3. Same volume effect occurs for other pairs, including twins such as Royal Dutch-Shell and stubs such as 3Com/Palm
4. Rule of thumb for all large pairs:
   - A one sigma increase in a pair's relative volume corresponds to roughly one-quarter of a sigma decrease in its relative price
5. There are likely about 20 distinct arbitrageurs who trade pairs.

Future Research

- Model Refinements and Generalizations
  - E.g. What if $\frac{V_E}{V_C} > 1$ only with probability *p*?
  - Empirical tests could be done across a wider variety of mispriced pairs that differ in their probability *p.*
- Relative Liquidity
  - What is its the theoretical relation with relative volume?
  - How do you measure and test in practice?
- Non-Stock Pairs (Bonds, Options, FX, …)
  - Does same effect hold? Same implied parameters?